Groundwater whirls in heterogeneous and anisotropic layered aquifers

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Abstract Groundwater flow is affected by heterogeneity in the anisotropy. Our investigations started with some simple numerical experiments consisting of one or two homogeneous anisotropic blocks. Three-dimensional pathlines in such models have the shape of spirals; bundles of spiralling pathlines were termed 'groundwater whirls'. The present research also deals with hypothetical models of stratified aquifers. Investigations are extended to include aquifers in which all layers have a laterally heterogeneous anisotropy. All models were built with the finite element model MicroFEM. In these tests the heterogeneity of the principal values of the hydraulic conductivity is not taken into account. Pathlines show complex patterns of clockwise and counter-clockwise groundwater whirls because of the heterogeneity in the anisotropy. The practical consequence of groundwater whirls is that water is exchanged between aquifer layers, even when the gradient of the hydraulic head in the aquifer as well as the general direction of flow is parallel to the layered structure of the aquifer. This may have a significant impact on contaminant spreading throughout the aquifer. Several analytic solutions have been developed that confirm the existence of groundwater whirls in anisotropic layered aquifers.
Key words layered aquifers; anisotropy; heterogeneity; finite-element model; MicroFEM; analytical model; pathlines; stream function

INTRODUCTION

Finite-element models were used to study how steady-state groundwater flow is affected by heterogeneity in the anisotropy. This topic has received little attention, in contrast to the abundance of groundwater literature on flow through heterogeneous but isotropic conductivity fields.

Our investigations started with some simple numerical experiments consisting of one or two homogeneous anisotropic blocks in an otherwise homogeneous and isotropic confined aquifer (Hemker *et al.*, 2004). Three-dimensional pathlines in such models have the shape of a spiral; bundles of spiralling pathlines turning in the same direction were termed 'groundwater whirls'. Pathlines would be straight if the layers are isotropic.

Here we deal again with hypothetical models of stratified aquifers, built with the finite-element model MicroFEM (MDS, 2004), but in this case all layers have a laterally heterogeneous anisotropy. Although in reality heterogeneity of the hydraulic conductivity will often play an important role, the present investigations were restricted to the effect of two-dimensional heterogeneity in the horizontal anisotropy. This means that all principal values of the hydraulic conductivity were chosen the same throughout the model, while only the major principal direction of the conductivity varies spatially in a plane perpendicular to the general flow direction. Such laterally discontinuous horizontal anisotropy leads to complex patterns of clockwise and counter-clockwise groundwater whirls. The results of MicroFEM have been confirmed with other numerical groundwater models such as SUTRA and MODFLOW.

New analytic solutions for groundwater flow in layered anisotropic aquifers have been developed recently (Bakker & Hemker, 2004) and allow for a comparison of the presented numerical flow patterns with the results of similar analytical models.

FINITE ELEMENT MODELLING

In this short paper only a single model is presented. Consider an 18 m thick confined aquifer, consisting of nine equally thick layers. The horizontal hydraulic conductivity is heterogeneous in a 100 m wide section of the model; each layer in this section is divided in 10 strips of equal width. A cross-section perpendicular to these strips shows a regular pattern of 9-by-10 cells, where each cell is 10 m wide and 2 m high (Fig. 1). On each side of this central zone a 100 m wide homogeneous block serves to reduce boundary effects.

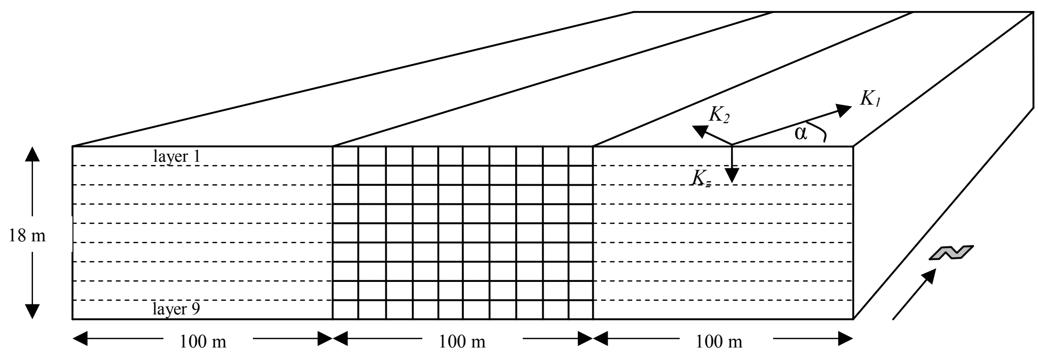

Fig. 1 A stratified confined aquifer with a laterally heterogeneous anisotropic central zone.

The major and minor principal values of the horizontal hydraulic conductivity tensor are 10 m day^{-1} and 5 m day^{-1} in the entire model. The vertical hydraulic conductivity is 1 m day^{-1} in all layers. The general flow direction is in the direction of the strips (straight north). In the presented model the major principal direction of the horizontal hydraulic conductivity tensor is also chosen straight north in the two large side blocks, while it varies between -45° (N45W, northwest) and 45° (N45E, northeast) in the 90 cells of the central zone. To obtain a two-dimensional spatial distribution, ten uniformly distributed anisotropy directions were chosen (-45°, -35°, -25°, …, 45°) and for each layer these ten values were assigned to the cells in a random order. The resulting distribution is given in Table 1.

To compute the three-dimensional head distribution and flow field, a finite-element model of 18 layers was built with MicroFEM (each 2 m thick layer is represented by two MicroFEM layers). The model size is 300 by 300 m; horizontal nodal distances are exactly 2 m within the central zone, and increase to nearly 10 m at the east and west model boundaries to attain a total of 11,668 nodes per layer. The east and west sides are no-flow boundaries, while the south and north sides are open boundaries with fixed potentiometric heads that differ by 0.3 m in all layers.

Table 1 Principal directions of horizontal anisotropy in all 9 by 12 cells of the model.

0	-35	5	15	35	-15	-25	-5	25	-45	45	0
0	25	35	-35	15	-45	-5	-25	5	-15	45	0
0	-35	25	-15	-25	-5	15	5	-45	45	35	0
0	-35	35	5	-25	15	-5	45	25	-15	-45	0
0	15	-5	25	-45	35	45	5	-35	-15	-25	0
0	-45	-35	25	-25	5	-15	15	35	-5	45	0
0	25	-45	-5	35	-25	5	15	-15	45	-35	0
0	35	5	-15	-35	25	-45	-25	15	45	-5	0
0	45	-25	-35	-5	-15	15	5	-45	35	25	0

NUMERICAL AND ANALYTICAL RESULTS

If the model would be isotropic or homogeneous in the anisotropy all pathlines would flow straight north and the projection of a single pathline, when looking downstream, would be a single point. The projection of 20 pathlines in each of the 9 layers of our anisotropic model is shown in Fig. 2. Pathlines start in the centre of each cell and at equal distances to the left and right in the homogeneous side blocks. All starting points are located at 100 m north of the southern boundary and all pathlines run for 100 m due north.

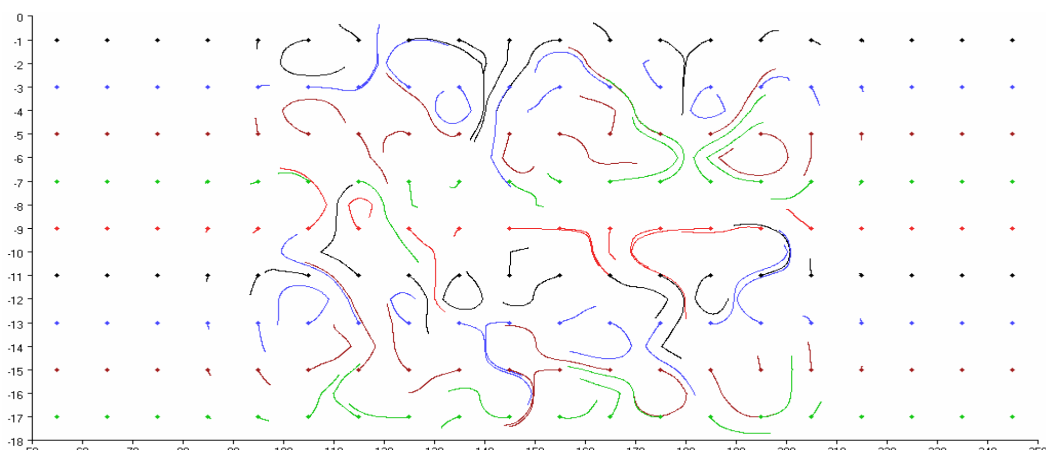

Fig. 2 Cross-section showing 9 * 20 projected pathlines to identify the pattern of clockwise and counter-clockwise whirls computed numerically with MicroFEM. Starting points are indicated with dots.

The resulting whirling flow pattern is complex and not immediately clear from a single graph. Large and small whirls exist next to each other, rotating in opposite directions. The slight extension of the whirls into the homogeneous side blocks is on the order of 20 m. The same pathlines of Fig. 2 are projected on a north-south cross-section (Fig. 3). Although different colours are used for different starting depth, just as in Fig. 2, individual pathlines are hard to distinguish. Therefore the same pathlines are given in two separate sets (Figs. 4 and 5). The vertical exchange of water between adjacent layers as a result of the whirling flow is clearly demonstrated.

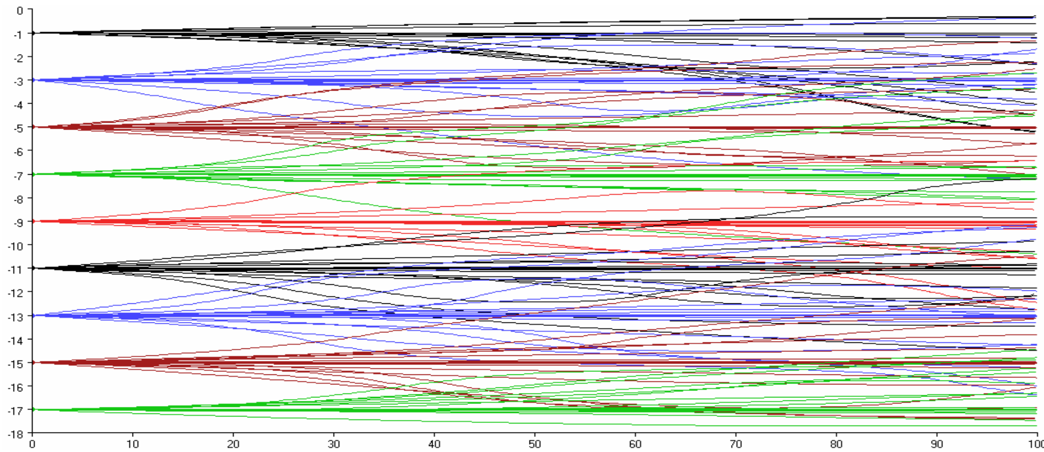

Fig. 3 Cross-section showing the 9 * 20 pathlines of Fig. 2 projected in a side view.

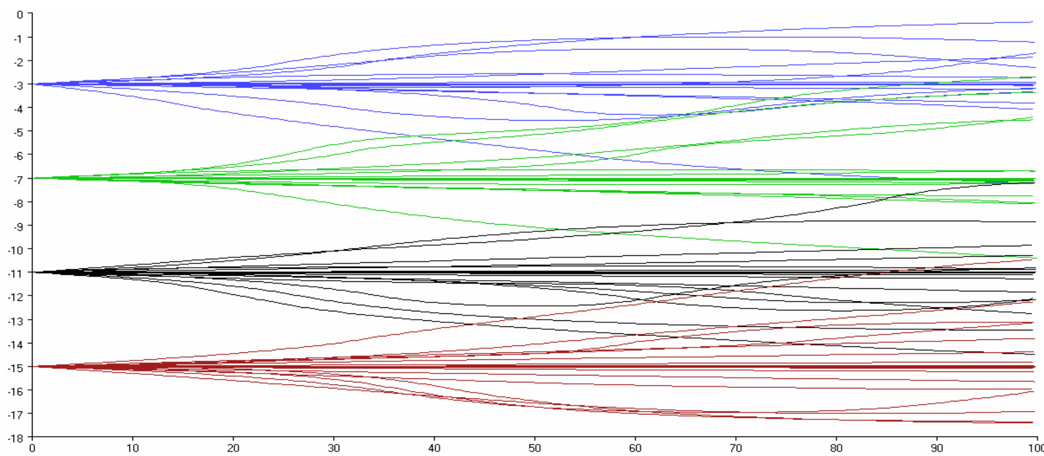

Fig. 4 Cross-section showing the 4 * 20 pathlines of Fig. 3 starting in the layers 2, 4, 6 and 8.

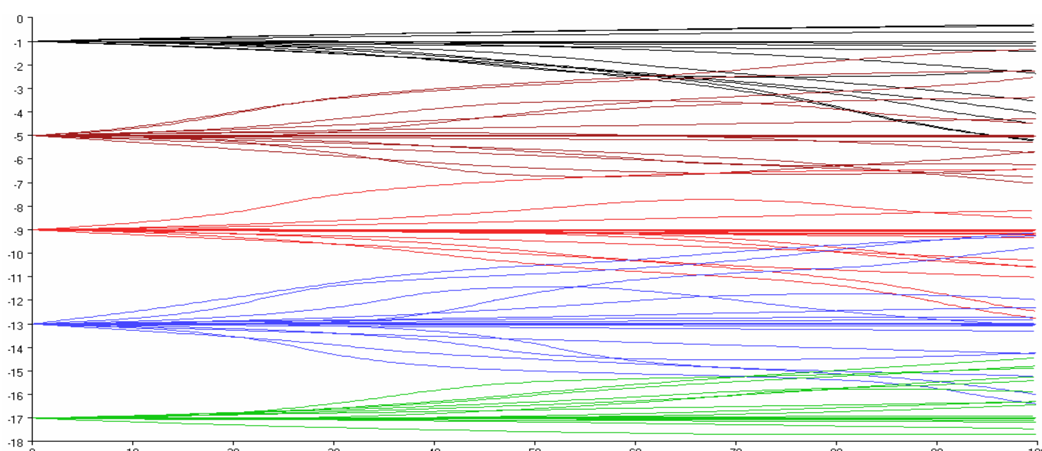

Fig. 5 Cross-section showing the 5 * 20 pathlines of Fig. 3 starting in the layers 1, 3, 5, 7 and 9.

New analytic solutions have been developed recently to compute the heads and flow in a layered anisotropic aquifer similar to the above described numerical model (Bakker & Hemker, 2004). The above described model is used here to compare the numerical results with the exact solution. The hydraulic head in all layers is shown in Fig. 6 and the east component of the specific discharge vector is given in Fig 7.

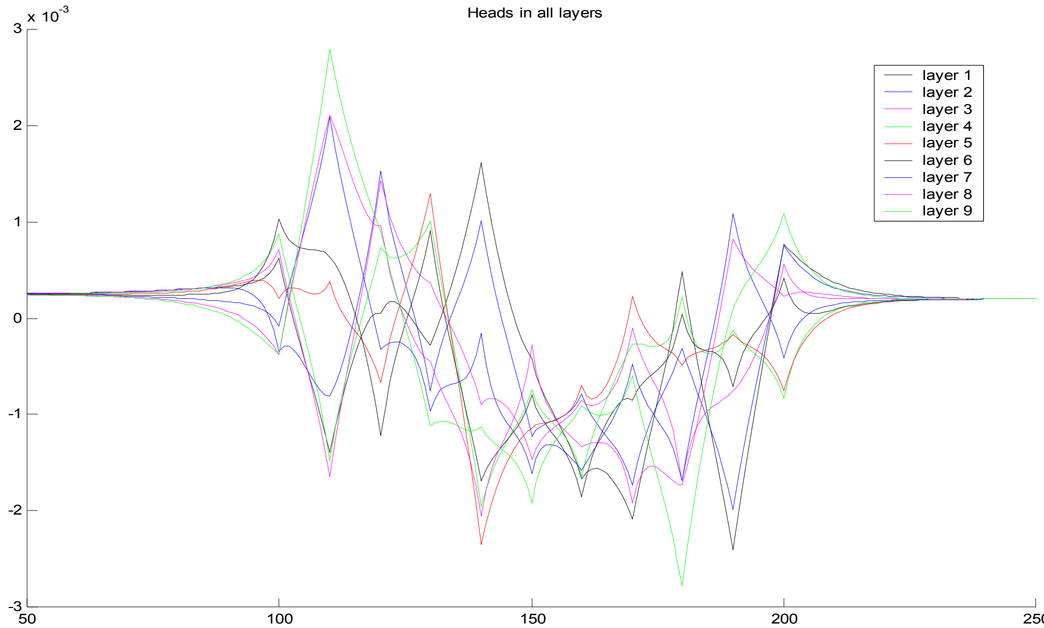

Fig. 6 Analytically computed hydraulic heads in an east-west cross-section of the layered aquifer.

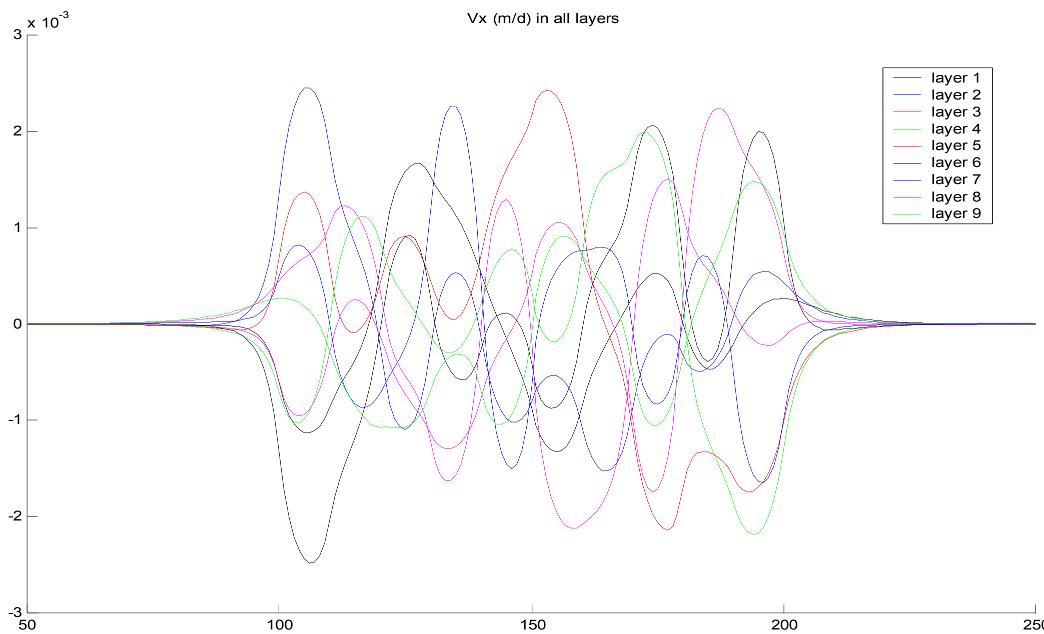

Fig. 7 Analytically computed lateral flow components in an east-west cross-section of the layered aquifer.

The analytical solution technique also allows for the computation of three-dimensional pathlines. A projection of such pathlines, starting at the same locations as as in Fig. 2 and also running for 100 m perpendicular to the cross-section, is presented

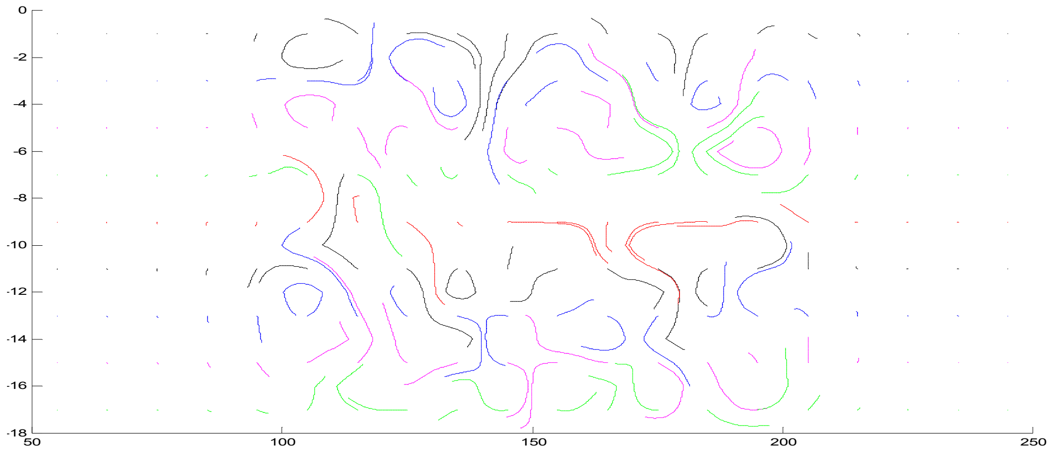

Fig. 8 Cross-section showing 9 * 20 projected pathlines computed analytically.

in Fig. 8. The similarity of Figs 2 and 8 is a verification of both the numerical model and the analytical model. The differences between the numerical and analytical results decrease if each aquifer layer is modelled with more MicroFEM layers.

Analytically obtained flow components in a cross-section can also be visualized as stream function contours (Fig. 9).

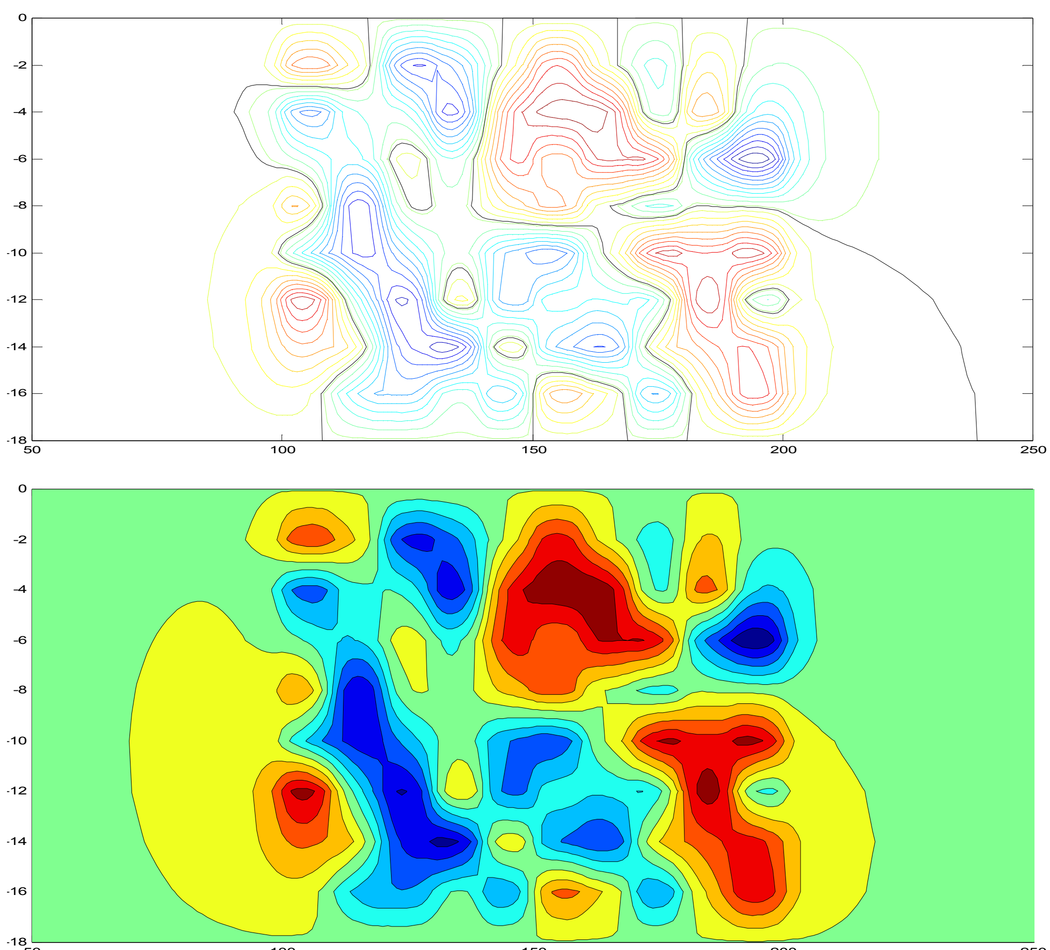

Fig. 9 Cross-section showing the pattern of clockwise (blue coloured) and counter-clockwise whirls (red coloured) by analytically computed stream function contours.

The analytical results (Figs. 6-9) confirm the whirling flow system as computed with three-dimensional pathlines in the MicroFEM model (Figs. 2-5). The detailed whirl pattern clearly shows that several smaller whirls may occur within larger whirls, all rotating in the same direction.

In the presented model the direction of the strips, the mean of the major principal anisotropy direction as well as the general hydraulic head gradient were chosen the same (straight north). The results of similar numerical and analytical models (e.g. Fig. 10) show that comparable whirl systems occur when these directions are chosen differently.

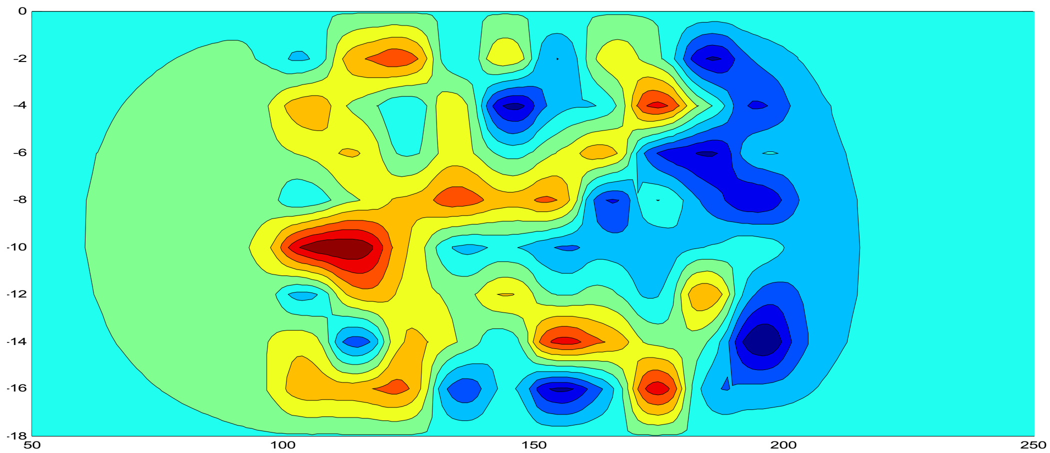

Fig. 10 Cross-section showing the pattern of clockwise (blue coloured) and counter-clockwise whirls (red coloured) when all principal anisotropy directions of Table 1 are rotated 45 degrees clockwise.

CONCLUSIONS

Pathlines in a steady-state stratified finite element model with heterogeneous anisotropy show complex patterns of clockwise and counter-clockwise groundwater whirls. These numerical results are confirmed by analytical solutions. The practical consequence of groundwater whirls is that water is exchanged between aquifer layers, even when the gradient of the hydraulic head in the aquifer as well as the general direction of flow is parallel to the layered structure of the aquifer. This may have a significant impact on contaminant spreading throughout the aquifer.

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